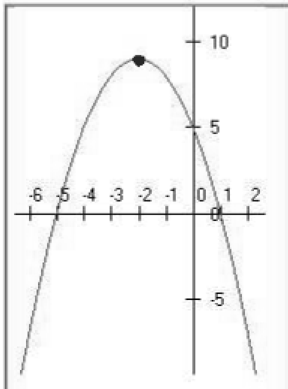
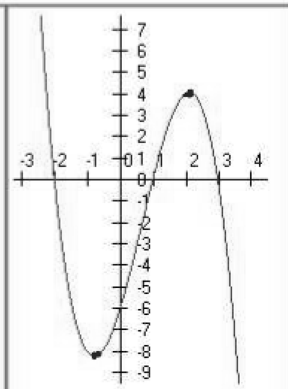
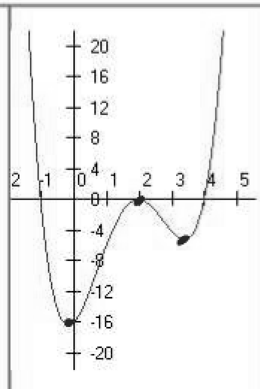
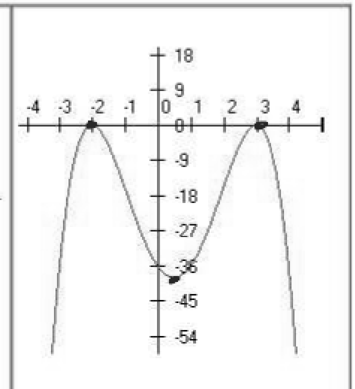
Multiplicity and Graphing Polynomials

SWBAT apply the multiplicity of roots; graph polynomial functions
Warm up

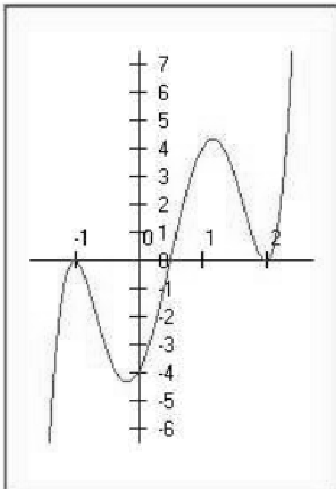
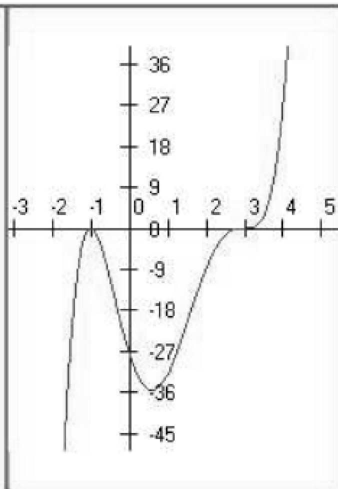
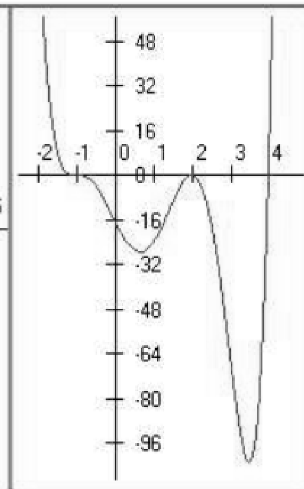
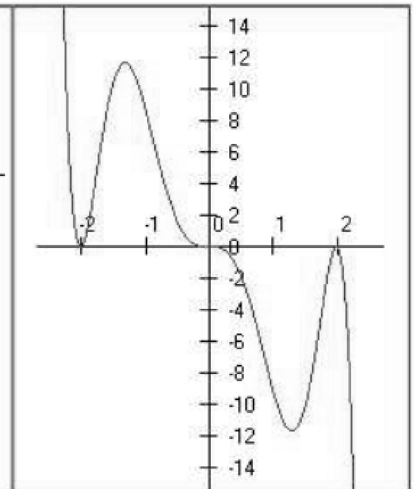
1) Describe the right-hand and left-hand behavior of the graph of the polynomial function.

$f(x) = -2x^5 - 5x^4 + 3x^3$ $\qquad\qquad$ $f(x) = x^4 - 29x^2 + 100$

2) LESSON EXPLORATION 

			
y =-(x-1)(x+5)	y =-(x-1)(x-3)(x+2)	y = (x+1)(x-2)^2(x-4)	y = -(x+2)^2(x-3)^2
Degree of Polynomial: 2	egree of Polynomial: 3	egree of Polynomial: 4	egree of Polynomial: 4
umber of Turning Points: 1	umber of Turning Points: 2	umber of Turning Points: 3	umber of Turning Points: 3

1) Do you see a relationship between the degree of the polynomial and the number of turning points that the graph can have?

			
$y = (x+1)^2(x-2)^2(2x-1)$	$y = (x+1)^2(x-3)^3$	$y = (x+1)^3(x-2)^2(x-4)$	$y = -x^3(x+2)^2(x-2)^2$

2) Look at the roots of each graph. Sometimes the graph crosses the x-axis and sometimes the graph "bounces off" the x –axis. Can you come with a generalization as to why this happens?

Real Zeros of Polynomial Functions

It can be shown that for a polynomial function f of degree n, the following statements are true.

1) The function f has, at most, <u>n real zeros</u>.

2) The graph of f has, at <u>most $n-1$ turning points</u>. (Turning points, also called relative minima or relative maxima, are points which the graph changes from increasing to decreasing or vice versa.)

If f is a polynomial function and a is a real number, the following statements are equivalent.

1) $x = a$ is a real zero of the function f.
2) $x = a$ is a solution of the polynomial equation $f(x) = 0$
3) $(x - a)$ is a factor of the polynomial $f(x)$.
4) $(a, 0)$ is an x-intercept of the graph of f.

Repeated Zeros
Definition: A factor $(x - a)^k$, $k > 1$, yields a **repeated zero** $x = a$ of **multiplicity** k.

1) If k is odd, the graph crosses the x-axis at $x = a$.

2) If k is even, the graph touches the x-axis (but does not cross the x-axis) at $x = a$.

Sketching Polynomials using zeros, end behavior, and the leading coefficient test.

To sketch the graph of a polynomial:
STEP 1: (a) Find the x-intercepts, if any, by solving the equation $f(x)=0$.
(b) Find the y-intercept by letting $x=0$ and finding the value of $f(0)$.
STEP 2: Determine whether the graph of f crosses or touches the x-axis at each x-intercept.
STEP 3: Determine the end behavior:
STEP 4: Determine the maximum number of turning points on the graph of f.
STEP 5: Use the x-intercept(s) to find the intervals on which the graph of f is above the x-axis and the intervals on which the graph is below the x-axis.
STEP 6: Plot the points obtained in Steps 1 and 5, and use the remaining information to connect them with a smooth, continuous curve.

For each of the following:

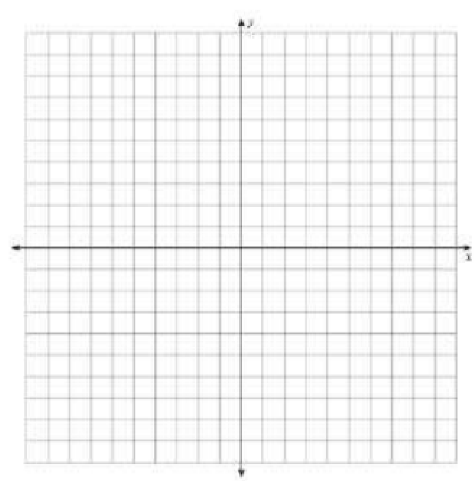
a) Describe the end behavior of the function.
b) Find all real zeros of the polynomial functions.
c) Determine the multiplicity of each zero and the number of turning points of the graph of the function.
d) Sketch the graph. Be sure to find the x and y intercepts

1) $f(x) = x^2 + 10x + 25$

a)

b)

c)



2) $f(x) = x^4 - x^3 - 20x^2$

$x^2(x^2 - x - 20)$

$x^2(x-5)(x+4)$

a) Zeros

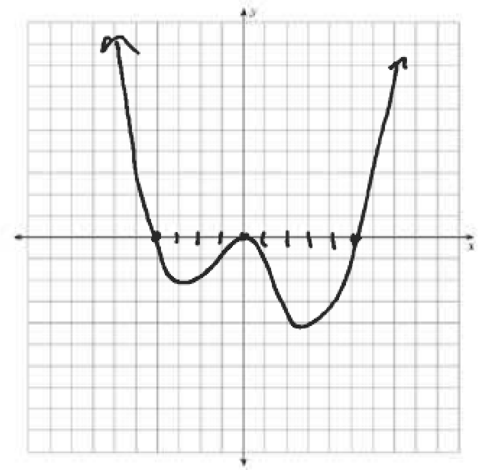
$x=0$ $x=5$ $x=-4$

mult 2 1 1

b)

y-int $(0,0)$

c)



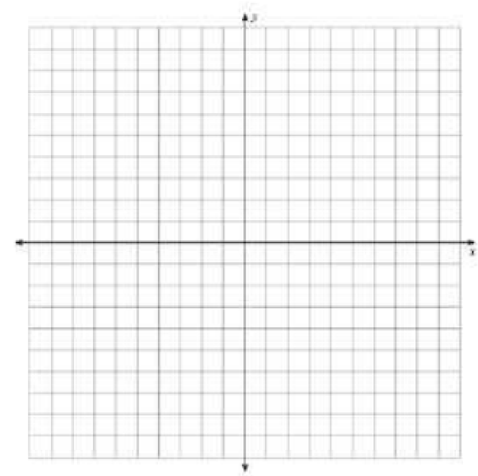
3)

$f(x) = -x^4 + 4x^3 - 4x^2$

a)

b)

c)



4)

$f(x) = (x^3 - 4x^2)(-25x + 100)$

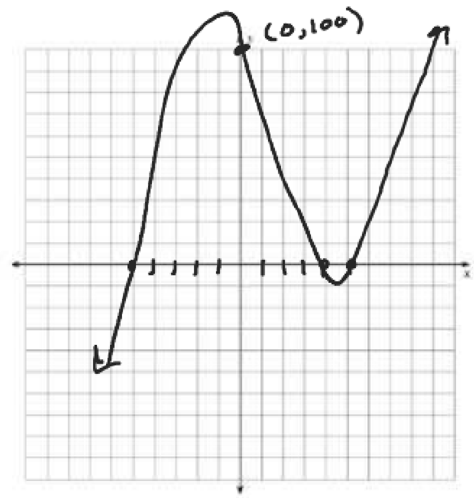
$x^2(x-4) - 25(x-4)$

a) $(x^2-25)(x-4)$

$(x+5)(x-5)(x-4)$

b)

c)



5) $f(x) = -(x+3)^3(x-2)^2$

$x=-3$ $x=2$

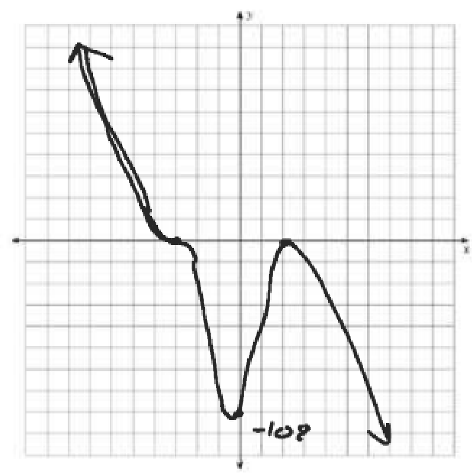
mult 3 mult 2

$(0,-108)$

a)

b)

c)



6) Refer to the graph below.

Write a possible equation for this polynomial in factored form.

$x = -\frac{5}{2}$

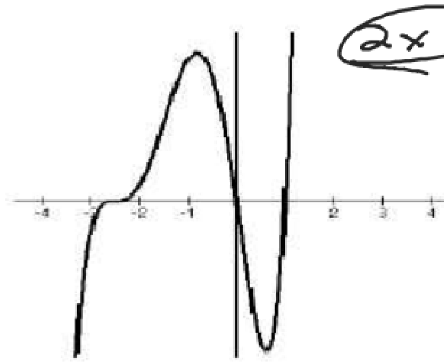
$2x = -5$

$2x+5 = 0$

$x = -\frac{5}{2} \qquad x = 0 \qquad x = 1$

$(x + \frac{5}{2})(x+0)(x-1)$

$y = x(2x+5)^3(x-1)$



7) Refer to the graph below.

Write a possible equation for this polynomial in factored form.